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J. Brandts, S. Korotov, M. Křížek, K. Segeth, J. Šístek, T. Vejchodský (Eds.)
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MY WONDERFUL NUMERICAL ANALYSIS TEACHERS — MILAN PRÁGER AND EMIL VITÁSEK

Michal Křížek

Institute of Mathematics, Academy of Sciences
Žitná 25, CZ – 115 67 Prague 1, Czech Republic
krizek@math.cas.cz

1. Numerical analysis at the Faculty of Mathematics and Physics

In 1970 I began to study mathematics at the Faculty of Mathematics and Physics at Charles University in Prague. In the third year, we had to choose one of the following specializations: algebra, mathematical analysis, applied mathematics, probability theory and statistics, topology, geometry, and numerical mathematics. My mother advised me at that time to choose numerical mathematics, since this was apparently a very new and modern discipline. I obeyed her suggestion, although I had absolutely no idea what this branch of science dealt with.

At the first numerical mathematics lecture Dr. Milan Práger showed us how to calculate the integral

$$I_n = \frac{1}{\mathrm{e}} \int_0^1 x^n \mathrm{e}^x \,\mathrm{d}x > 0. \tag{1}$$

First, using integration by parts, he derived the recurrence formula (cf. [14, p. 505])

$$I_n = 1 - nI_{n-1}, \quad n = 1, 2, \ldots, \tag{2}$$

and then he said that for simplicity we will evaluate the individual integrals only to three decimal places. Gradually he calculated on the blackboard the following values:

$$I_0 = 1 - \mathrm{e}^{-1} = 0.632, \qquad I_1 = 1 - 0.632 = 0.368, \qquad I_2 = 1 - 2 \cdot 0.368 = 0.264,$$
$$I_3 = 1 - 3 \cdot 0.264 = 0.208, \quad I_4 = 1 - 4 \cdot 0.208 = 0.168, \quad I_5 = 1 - 5 \cdot 0.168 = 0.16,$$
$$I_6 = 1 - 6 \cdot 0.16 = 0.04, \qquad I_7 = 1 - 7 \cdot 0.04 = 0.72.$$

Slowly I began to get bored and in my mind I wondered: *That, that is the modern mathematical discipline?* Then a big surprise came. Dr. Práger calculated

$$I_8 = 1 - 8 \cdot 0.72 = -4.76$$

and said: *Notice, dear students, that we have got a negative value, while the integral* (1) *is certainly positive. This is a completely unacceptable numerical result.* I immediately thought that the absurd negative number must be just a result of rounding errors, and I began to suspect what numerical analysis is about. At that time, of course, I had no idea about the instability of scheme (2) that was examined by Renata Babušková in her 1964 paper [5] (cf. also [1, p. 102]).

The above numerical phenomenon happens due to the fact that at each step we subtract two numbers of almost the same size. Then the difference contains only a few nonzero significant digits in computer arithmetic that necessarily leads to loss of accuracy. A very similar recurrence to (2) was examined by Muller [7].

I do also remember very well my first seminar on numerical mathematics. With Dr. Jitka Segethová we calculated the values of polynomials using Horner's scheme on large and heavy mechanical calculators that were powered by an electrical engine. Nevertheless, on the ground floor of our building on the Lesser Town Square there already was a big mainframe electronic computer Minsk 22. Here I used ALGOL 60 (Algorithmic Language) to program simple numerical algorithms that Dr. Práger taught us. Minsk 22 had 64 KB of memory, input via punched tape, and was very slow. Moreover, approximately every 20 minutes computer calculations crashed due to MACHINE ERROR. So basically it was not possible to perform any longer calculation. We learned also the machine code to speed up computations.

In the fourth year of my studies, the numerical mathematics was taught by Dr. Emil Vitásek. In fact, the recurrence (2) was invented by him (see [1]). He lectured by heart using no written notes and with great enthusiasm. His performance was truly wonderful, logically assembled, and understandable. He concentrated on solving partial differential equations by the finite difference method, which is a sort of forerunner of my favorite finite element method. In particular, I was charmed by the convergence proof of the finite difference method that he presented to us.

2. Department of Constructive Methods of Mathematical Analysis

During my military service in 1975–1976, I received a letter initiated by Dr. Práger, whether I wanted to start postgraduate studies at the Mathematical Institute of the Czechoslovak Academy of Sciences. Because I had not negotiated any further job after completing my military service, I agreed, and certainly at present I do not regret that decision. Therefore, in September 1976 I began postgraduate studies with Dr. Práger at the Department of Constructive Methods of Mathematical Analysis, where he was the Head during the period 1969–1994. His Deputy was Dr. Vitásek. The Department was located at the rear of the Opletalova street no. 45. Dr. Práger and Dr. Vitásek shared the front room, where also our Numerical Analysis Seminars

were held. I did not understand the first several lectures there and I have to admit that it took me quite a long time to follow the issues that were investigated in our Department. I started to read at that time the recent paper [12] on overimplicit multistep methods for ordinary differential equations written by M. Práger, J. Taufer, and E. Vitásek.

For my Candidate of Sciences examination I studied the classical 1966 monograph *Numerical processes in differential equations* [3] by Ivo Babuška, Milan Práger, and Emil Vitásek. It already contained the description of the finite element method for elliptic boundary value problems — my favorite topic. Contour lines of the standard piecewise linear finite element basis functions are illustrated in [3, p. 305]. This picture serves, in fact, as the LOGO of our Numerical Analysis Seminar and also of this Conference. Some other linear and bilinear finite element basis functions were already sketched in their previous book [2] published in the Czech language.

Both the monographs [2] and [3] begin with the recurrence (2). However, there are other nice and illustrative numerical examples — for instance, the investigation of numerical instability of successive performance of the following arithmetic operations

$$\ldots(((((1:2)\cdot 2):3)\cdot 3):4)\cdot 4\ldots$$

Various numerical results of this expression were obtained by Karel Segeth on different computers involving thousands of divisions and multiplications [3, p. 6]. I liked such examples very much. Later I wrote the article [6] jointly with M. Práger and E. Vitásek on the reliability of numerical computations. We systematically collected many other pathological examples, where the numerical solution behaves in an unpredictable manner. This resulted in another article [19] with Dr. Vitásek and I continue with this activity ever today. The main reason is that programmers should not always believe their computer outputs, in particular, if they are not familiar with numerical analysis topics like finite precision arithmetic, theory of rounding errors, ill-conditioned problems, and so on.

In our Department there has always been a great friendly and creative atmosphere. I can begin to describe what I have learned from my numerical analysis teachers in over 40 years. Dr. Milan Práger significantly contributed to the issue of numerical modelling in electrical engineering. Together we dealt with several real-life technical problems for the Research Institute VÚSE Běchovice. In particular, Dr. Práger numerically calculated the magnetic field inside large oil-immersed transformers. Then I used his results on the density of heat sources to calculate the temperature distribution in the magnetic core. With Dr. Vitásek I discussed the various theoretical aspects of numerical methods that I applied. He wrote a whole series of monographs (see [2], [3], [4], [16], [17]) devoted to numerical methods. They originated from our Department in Opletalova street without any personal computers and internet.

Allow me to finish this section by a funny story. A Vietnamese aspirant once visited our Department and was looking for Dr. Vitásek. Dr. Práger told him that

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Dr. Vitásek is lecturing in Italy and will return after three weeks. However, the Vietnamese aspirant did not understand well, he leaned in a large armchair and said: *Never mind, I'll wait.*

3. Milan Práger — Curriculum vitae

RNDr. Milan Práger, CSc., was born on April 21, 1930 in Prague. After grammar school in Smíchov in 1940–1948 he became a student of mathematics at the Faculty of Science of Charles University in Prague. His studies ended in 1952 when he passed the leaving State Examination. During the period 1952–1954 he worked at the Faculty of Mechanical Engineering of the Czech Technical University in Prague as an assistant in the Mathematical Department, and then became a postgraduate student of Ing. Dr. Ivo Babuška at the Mathematical Institute of the Czechoslovak Academy of Sciences in Prague (1954–1957). He received the scientific title Candidate of Sciences (CSc. $\approx$ PhD.) in the year 1959, and stayed to work at the Mathematical Institute as a researcher, senior researcher (1965), and chief researcher (1977). From 1971 to 1992 he was the Head of the Department of Constructive Methods of Mathematical Analysis. He retired in 1996, but still worked part-time at the Mathematical Institute until 2005. During this period he published several valuable papers, see e.g. [9], [10], [11]. He is still interested and participates in our regular Friday seminar *Current problems in numerical analysis.*

The main subject of scientific interest of Dr. Prager is the theory of numerical methods for solving differential equations. He has published about 40 original mathematical papers, conference articles, co-authored the 1964 monograph[1] *Numerical*

[1]The logo from the front page of this Proceedings was taken from [3, p. 305]. It shows the support of a linear finite element basis function with its contour lines.

Dr. Milan Práger lecturing at our Numerical Analysis Seminar

solutions of differential equations [2] and the 1966 monograph *Numerical processes in differential equations* [3], which was translated into Russian in 1969, see [4]. He also wrote a chapter in the world wide known Rektorys' *Survey of applicable mathematics* [14], which was published in two English and six Czech edition series. Another important accomplishment is his textbook *Numerical mathematics I* [8].

Dr. Práger participated in numerous domestic and international scientific meetings and research visits. Let us name, for instance, a few series of Equadiff Conferences. He was also a lecturer at the postgraduate course at the University of Zagreb, the Istituto per le Applicazioni del Calcolo in Rome, at Chalmers University of Technology in Gothenburg, at the Royal Institute of Technology in Stockholm and in many other places in former Czechoslovakia.

In addition, Dr. Práger was always intensely concentrated on educational activities and organization of scientific meetings. During the period 1967–1990 he taught fundamentals of numerical methods at the Faculty of Mathematics and Physics of Charles University in Prague. He is the author of lecture notes on this topic and translated with Dr. Emil Vitásek the comprehensive Ralston's guide *A first course of numerical analysis* [13]. Dr. Práger was the advisor of my Candidate of Sciences thesis *An equilibrium finite element method in three-dimensional elasticity* defended in 1980. Its co-advisor was his colleague Ivan Hlaváček. Dr. Práger led theses of other four scientific aspirants: Michal Kočvara, Stanislav Míka, Karel Višňák, Jan Vlček, and successfully trained several master students.

Milan Práger was a member of the final state examination committee at Charles

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University in Prague, a member of the committee for candidate and doctoral dissertations, a member of the National Commission on issues of information technology, and many others. He was also a co-organizer of more than ten years of popular summer school "Programs and Numerical Algorithms" traditionally held at various locations of Jizera Mountains.

It would take a long time to enumerate what Milan Práger has done for mathematics and for the Institute of Mathematics. Let us mention also his interests that go far beyond mathematics. For example, he has a deep knowledge about history, cartography and music, he likes to solve various puzzles and cross-words, he is a very good chess player and played for many years in the chess section of Prague universities.

4. Emil Vitásek — Curriculum vitae

RNDr. Emil Vitásek, CSc., was born on May 29, 1931 in České Budějovice. After high school in Přerov, where he graduated in 1950, he began to study mathematics at the Faculty of Science of Charles University, which in 1953 changed to the Faculty of Mathematics and Physics. After his studies he joined the Mathematical Institute of the Czechoslovak Academy of Sciences in 1954 as a research assistant in the department of Ing. Dr. Ivo Babuška. Under Dr. Babuška's leadership, he attained the Candidate of Sciences degree CSc. in 1960. Dr. Vitásek became a researcher and later a senior researcher. He is still actively working at the Institute of Mathematics (see e.g. [18]).

Dr. Emil Vitásek lecturing at our Numerical Analysis Seminar

His mathematical research is associated with the numerical solution of differential equations, in particular, numerical methods for time-dependent equations, i.e. ordinary and parabolic equations. There he employed his deep knowledge of mathematical and functional analysis. His first papers are associated with the calculations of the Dam Orlík on the Vltava river. Then he dedicated himself to the study of numerical stability. He was one of those who developed the theory of transfer boundary conditions for boundary value problems for ordinary differential equations. At the same time he dealt with problems associated with engineering practice. He published about 60 original scientific papers and held lecture courses in Croatia, Sweden, and Italy. He was invited to give plenary lectures at several national and international conferences.

Dr. Vitásek is a member of the Editorial Board of Applications of Mathematics since 1971. He is the author of a chapter in the *Survey of applied mathematics*. He contributed three chapters to its last edition [14] and was the Associate Editor of its two volumes. He is also a co-author of the book *Numerical solutions of differential*

equations (1964), which has been revised and expanded to the English version Numerical processes in differential equations (1966) and was published in 1969 and in Russian translation (see [2], [3], [4]).

We should also mention the long-term pedagogical activity of Dr. Vitásek. He lectured on Numerical mathematics at the Faculty of Mathematics and Physics for more than 20 years, and then at the University of West Bohemia in Pilsen. He was the advisor of several Master students and four PhD students: Ľubor Malina, Hassan Nasr, Jan Šafář, Jiří Taufer. His fifth student Marian Brezina defended PhD in USA. In connection with these activities two of his monographs in the field of numerical mathematics appeared: *Numerical methods* [16] and *Foundations of the theory of numerical methods for solving differential equations* [17]. Another important accomplishment are his three textbooks. The first one *Numerical mathematics II — Numerical solution of differential equations* [15] was published by Charles University. The other two were published by the University of West Bohemia: *Selected chapters from the theory of numerical methods for the solution of differential equations* and *Introduction to the theory of generalized functions* that discusses the foundations of the theory of distributions. He also translated with Dr. Práger the famous monograph by A. Ralston: *A first course in numerical analysis* [13]. Dr. Vitásek was a member of the board of examiners for the Final State Exams and the board for the Rigorous Exams.

Emil Vitásek is a researcher with wide interests connected mainly with technical problems. He has a deep knowledge of aviation, but also of modern history and literature. He won the Czechoslovak national championship in correspondence chess. Anyone who comes to him with any problem, whether mathematical or generally human, finds that he is always a patient and attentive listener. Finally, we would like to mention his continuous aversion against the communist regime.

5. Felicitations

We all wish to Dr. Milan Práger and Dr. Emil Vitásek to their jubilees a good health and great satisfaction for a number of happy years.

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